

Hilbert's 19th Problem

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Abstract.

In this seminar we will discuss Hilbert's nineteenth problem, which asks whether minima for compact variations of regular variational problems are always smooth. By regular problems we mean variational problems of minima, defined on domains with Lipschitzian boundary, with smooth and uniformly convex Lagrangian function. We shall see how this problem leads back to the proof that weak solutions of uniformly elliptic linear PDEs in divergence form with bounded and measurable coefficients have a certain Hölder-regularity. This deep result was independently achieved by Ennio De Giorgi and John Nash in 1957, and provides a positive answer to the original problem. We will follow De Giorgi's approach.

References

- [1] Xavier Fernández-Real, Xavier Ros-Oton, *Regularity Theory for Elliptic PDE*.